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Comparing Basal Area Growth Rates in Repeated Inventories: Simpson's Paradox in Forestry

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ABSTRACT. Recent analyses of radial growth rates in southern commercial forests have shown that current rates are lower than past rates when compared diameter class by diameter class. These results have been interpreted as an indication that the growth rate of the forest is declining. In this paper, growth rates of forest populations in Alabama are studied. Basal area growth (a function of both radius and radial growth squared) by diameter classes is examined for plantation and natural stands. Basal area growth and population distributions for the 1962–1972 and 1972–1982 measurement periods are presented. Also, significance of Simpson's paradox in these analyses is discussed. Basal area growth proves to be consistent with changes that have occurred in tree frequency in diameter classes, i.e., stand structure. In Alabama's natural stands, basal area growth is shown to be relatively constant over the most recent inventories, while it has increased in plantations. Simple comparison of radial growth would be misleading. FOR. SCI. 35(4):1029–1039.

ADDITIONAL KEY WORDS. Contingency table, diameter distribution, radial growth, unequal probability sampling, sample stratification.

MUCH PUBLIC ATTENTION HAS BEEN GIVEN to radial growth rates and possible extensive declines in the growth rate of southern commercial forests (Sheffield et al. 1985). Unpublished analyses of radial growth rates in survey units in Alabama showed a pattern that creates the suspicion that widespread growth declines might be occurring in Alabama (Figure 1). These initial analyses for Alabama have been carefully reviewed for statistical validity of the broad interpretation given them and additional analyses performed that contradict the original interpretation.

The following problems were identified in the original analysis of the Alabama radial growth data:

1. Simple means of radial growth by diameter class were computed for data that were collected using point sampling, an unequal probability sampling scheme. Although estimation by diameter class removes some of the need for considering the unequal probability selection, recent research has shown that significant bias results when unweighted means are substituted for the probability weighted estimates of radial growth (Lappi and Bailey 1987, Lynch 1988).
2. Radial growth seems less appropriate than basal area growth as a comparative measure of growth rates by diameter class. Ultimately, we are interested in volume growth, and basal area growth is more closely related to volume growth than is radial. For a given radial growth, there are large differences in basal area growth for trees with different initial diameters especially in small trees. Therefore, direct comparison of basal area growth among diameter classes is clearer than comparisons based on radial growth. Transformation of radial growth to basal area growth does not alter the order of diameter class means between

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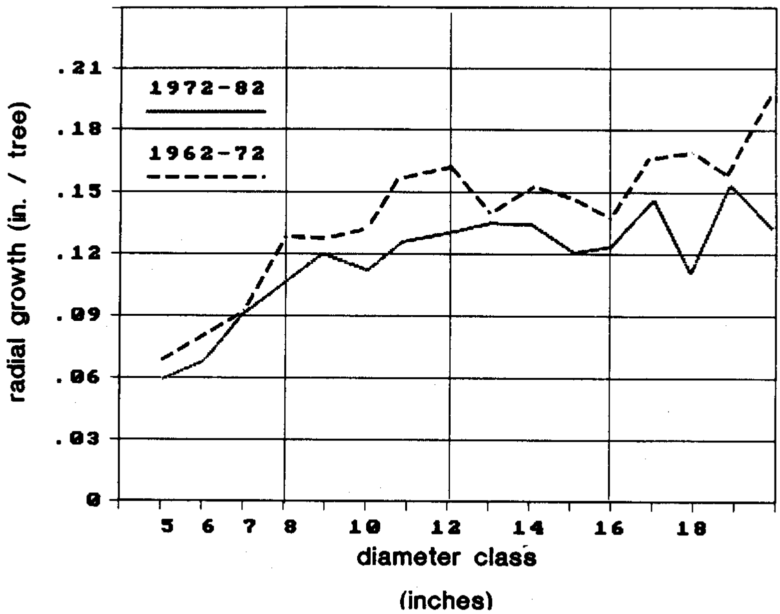

FIGURE 1. Radial growth rate by diameter class for natural loblolly pine, Alabama, Unit 4.

populations, i.e., if the mean radial growth is greater for the first population, the mean basal area growth for the first population will also be greater. For these reasons, we chose to examine the probability weighted basal area growth of survivor trees, instead of simple radial growth, thereby accounting for both diameter distribution and growth changes.

3. Simpson's paradox (Cohen 1986) may apply in the comparison of the growth rates by diameter class in consecutive time periods. Changes in the diameter class frequencies of the underlying population cannot be ignored. The paradox can be resolved by recognizing these differences in diameter distribution of past and current tree populations and by realizing that we are interested in changes in the growth of populations as a whole at least as much as individual trees. Foresters recognize that dominant 5-in. dbh trees that are leading a wave of reproduction in the South grow at a considerably faster pace than suppressed 5-in. trees growing in the wake of a wave of maturing poletimber.

In this paper we are primarily concerned with Simpson's paradox. The radial growth-basal area growth issues are mentioned because focus is shifted from radial to basal area growth and we feel recent literature and comparability among diameter classes favor analysis of basal area growth.

SIMPSON'S PARADOX

Cohen (1986) describes the uncertainty principle associated with Simpson's paradox that arises when two populations are stratified. Usually, the stratification process is undertaken to increase homogeneity among the strata. However, stratification may reverse the apparent rank ordering of the two population means. This phenomenon has been known to statisticians for over 50 years. Resolution of the paradox is dependent on recognizing the effect of changing frequencies among strata that are to be compared.

Cohen describes a situation in which the overall mortality rate for the period 1958–62 (crude death rate) was less for the female population of Costa Rica than for the female population of Sweden (Figure A1, Appendix). Intuitively, there should have been at least one age stratum in Sweden's population that had higher mortality than its corresponding stratum in Costa Rica's population. The figure also shows that this intuition is false; mortality rate at each age was higher for Costa Rica than for Sweden. Figure A2 (Appendix), also from Cohen, depicts the population frequency distributions for the two countries. The reason for Simpson's paradox is demonstrated by the difference in these two frequency distributions. The high frequency in high mortality rate classes in Sweden places more weight on these classes; therefore, the overall mortality rate for Sweden is higher than Costa Rica. Other studies (e.g., Shapiro 1982) of Simpson's paradox make it clear that comparisons of rates or measurements that are weighted averages from strata (or subgroups) are especially susceptible to misinterpretations resulting from the paradox. In forestry, mortality rate estimates are another susceptible statistic.

For a simple contingency table example in a forestry context, examine the values in Table 1. This hypothetical example shows the classification of 20 stands into 2 strata, pole-size stands and sawtimber-size stands. Two time periods are represented, a previous survey and a recent survey, and a measure of average growth is shown for each stratum in each time period. In the first survey, growth rates were higher for both sawtimber and poletimber (in both cases stands in the earlier period might well be younger than the current stands, thereby accounting for higher growth rates in the earlier period). It would be false, however, to say that the more recent growth is less than previous growth, because in aggregate we see that overall growth for the most recent period is $(16 \times 75 + 4 \times 55) = 1420$, and evaluation of the same expression for the previous period is 1400. This seeming paradox results from the shift in distribution of the stands from the previous survey to the more recent survey and not from any fundamental decline in tree growth. For this example, we can assume the shift is due to the cutting in sawtimber stands that removed the larger mature trees, resulting in six stands being reclassified in the more recent survey.

Simpson's paradox does not necessarily occur. Cohen notes that if the underlying population distributions are identical and every stratum in population B exceeds population A, then the intuition regarding overall rates must be correct. This simply emphasizes that the analyst must examine both the properly weighted strata means and relate them to their respective population frequency.

METHODS

The data consist of independent plots represented by the 1972 and the 1982 remeasurements of Alabama survey units (Figure 2). Data from both natural stands and plantations were analyzed. Two separate population analyses for

TABLE 1. Contingency table example of Simpson's paradox relative to forestry.

		Stand size classification	
Survey period		Pole	Sawtimber
Recent	frequency (no. stands)	16	4
	growth (ft^3/ac)	75	55
Previous	frequency (no. stands)	10	10
	growth (ft^3/ac)	80	60

natural and plantation stands (that may help to reassure those concerned as to the reasonability of current growth rates) will be useful for illustrating some of the characteristics associated with Simpson's paradox. Four units were selected to represent different population shift patterns (Figure 2). The Southwest-North unit (2) showed little change in population frequency by diameter class. The North Central unit (5) showed large differences in population distribution within plantations. Separate analyses of plantation and natural stands within the survey units serve to illustrate the major shift in interpretation when population structure shifts (i.e., is dynamic).

Probability weighted[1] mean basal area growth and total tree frequency of the population distribution for each 1-in. diameter class were calculated using individual trees within classes 5-in. and above. The following equation expresses the weighted mean basal area growth per tree:

$$g_{ba} = \Sigma w_{i1}(ba_{i2} - ba_{i1})/\Sigma w_{i1} \tag{1}$$

where: $w_{i1} = BAF/ba_{i1}$, the initial selection probability weight (also the tree's representative factor), BAF represents the basal area sampling factor, ba_{i1} the initial basal area, and ba_{i2} the terminal basal area. The total number of trees is represented by Σw_{i1}. Standard errors of estimate for the diameter and basal area growth were also computed using weighted equations.

RESULTS

Table 2 consists of basal area growth rates per acre and standard errors for natural and plantation stands over two growth periods. Note that the comparison is within plantations and natural stands and not between them; the analyses are the same for each of these stand conditions. Basal area growth of natural stands was essentially unchanged between periods. Plantation conditions show increased growth in Southwest-North and North Central (units 2 and 5), no change in West Central (unit 4), and a decline in Southeast (unit 3). All means are based on relatively large sample sizes, the distributions of means is approximately normal, so the half width of a 67% confidence interval is approximately equivalent to one standard error. Based on these standard errors, it is clear by inspection that none of the means are significantly different from one period to the next.

Table 3 presents the quadratic diameter means and associated standard errors for the two periods for natural and plantation sites in the four survey units. In general, changes in quadratic mean diameter for the period indicate increasing size of trees in both natural and plantation stands. The exception is Southwest-North (unit 2), which shows a decline in average diameter in natural stands and virtually no change in plantations.

In Figure 3, mean basal area growth per tree is plotted against diameter class for loblolly plantation data from unit 5. Nine of the 12 corresponding diameter class growth rates for the 1962–72 period exceed those for the more recent period. The wild swings in basal area growth rate in the diameter classes above 15-in. are due to small numbers of observations typical of inventory data. Examination of population frequency by diameter cells (Figure 4) shows that there was a shift in the distribution. There are many more

[1] The estimation is weighted by the inclusion probability of sample trees. Standard errors are based on the probability sample (Cochran 1977). Each tree receives its individual probability; results are not the product of the class mean diameter and growth, hence within-class distributions are completely accounted for.

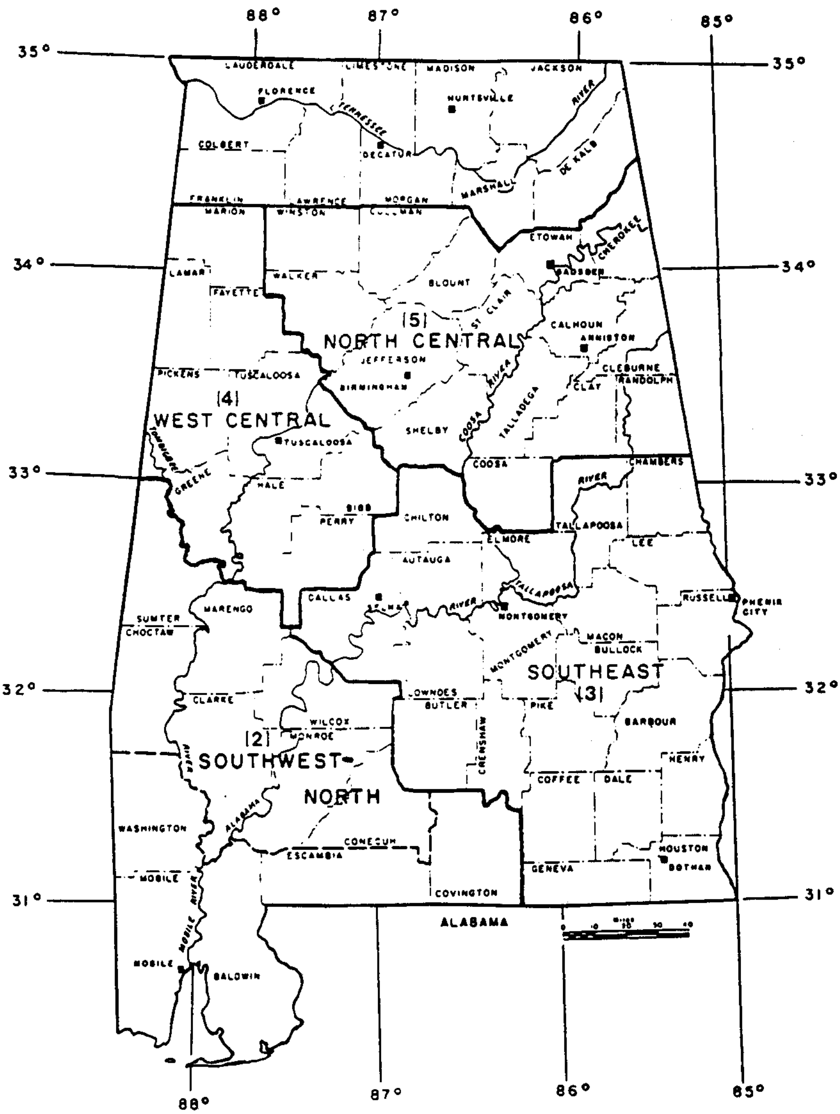

FIGURE 2. Survey units in Alabama.

trees from the 1982 survey in the 7–10 in. diameter classes than from the 1972 survey. This distributional shift is responsible for the increase in overall basal area growth rate from 22.8 ft^2/ac in 1972 to 26.5 ft^2/ac in 1982. One would not have concluded that an increase had occurred from inspecting the unweighted rates shown in Figure 3.

Another example, this time from natural stands, illustrates that mean reversal does not occur under all circumstances. As mentioned previously, Cohen proves that if there is no change in population frequency, then there can be no exchange in the order due to disaggregation. Furthermore, for some small shifts in frequency the reordering of means would not usually occur. Figures 5 and 6 represent the best counter-example available in the data. Throughout Alabama there was a trend to older, more mature stands of larger average diameter and density, so that most frequency distributions shifted to the right (Rudis et al. 1984). Natural stands of loblolly in unit 2

TABLE 2. Periodic basal area growth rates for loblolly pine by stand type in selected survey units in Alabama.

Survey unit	Natural stands				Plantations			
	1962–72		1972–82		1962–72		1972–82	
	Mean	SE	Mean	SE	Mean	SE	Mean	SE
	basal area ft²/ac/decade							
2	30.1	4.0	29.5	3.7	23.1	3.4	27.9	4.0
3	24.9	2.8	26.7	3.5	24.9	2.8	21.9	2.7
4	25.9	3.8	26.3	3.1	22.8	2.7	22.9	3.4
5	25.1	3.1	25.6	3.0	22.8	4.0	26.5	4.0

showed little change in mean basal area growth rates per tree for the smallest diameter classes, with declines apparent only in the 10–20-in range (Figure 5). Here the frequency distribution increased for the smaller diameter classes during the most recent survey, but little change occurred in the larger diameter classes. Hence, even though there were frequency shifts, they were insufficient and in the wrong part of the diameter distribution to produce the Simpson paradox effect. Table 2 shows that in unit 2 there remained a small decline in the basal area growth rate of natural stands (still *not* statistically significant).

For natural stands of the principal pine species (loblolly) in the units illustrated (as well as in other survey units in Alabama), overall mean basal area growth rate per tree remained essentially unchanged between the two survey periods. The examples also show that basal area growth rates in plantations had nonsignificant increases between the two measurements, even though basal area growth rate per tree by diameter class was down (Figures 3 and 4). These correctly weighted findings certainly corroborate industry expectations of plantation management, and the proper statistical analyses reassure us that Alabama's commercial forest growth rates are not declining.

Simpson's paradox is a key to understanding the apparent discrepancy between declines in basal area growth per tree and performance of stand level statistics. Intuitive comparisons of per tree growth rates over time, when broken into diameter class strata, may be misleading. Essentially, foresters must always recognize the effect that shifting diameter distributions may have on the forest level growth. Like mortality rates stratified by

TABLE 3. Quadratic mean diameter of loblolly pine by stand type in selected survey units in Alabama.

Survey unit	Natural stands				Plantations			
	1962–72		1972–82		1962–72		1972–82	
	Mean	SE	Mean	SE	Mean	SE	Mean	SE
	in. at breast height							
2	8.96	0.40	8.68	0.20	7.39	0.65	7.43	0.72
3	8.26	0.16	8.82	0.33	7.12	0.25	7.40	0.50
4	7.86	0.23	8.68	0.41	6.98	0.31	7.57	0.55
5	7.76	0.17	8.23	0.31	6.86	0.45	7.99	0.68

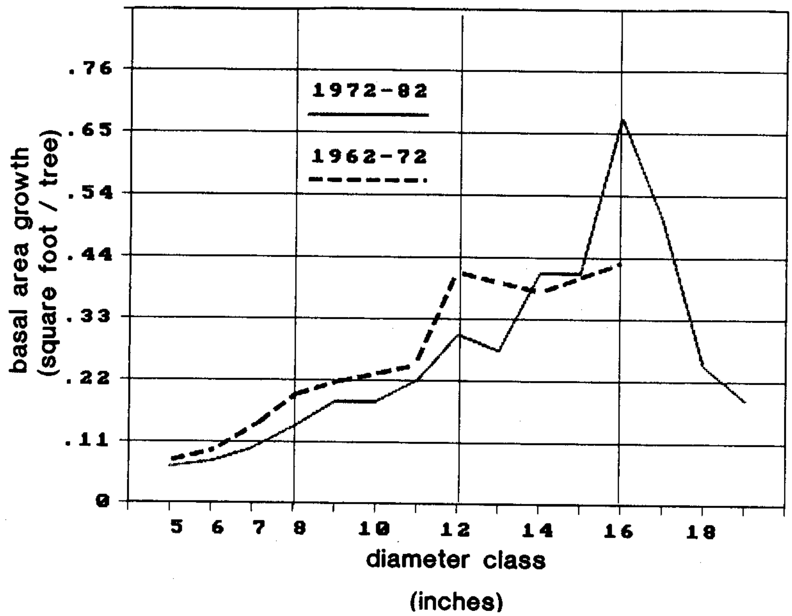

FIGURE 3. Basal area growth per tree by diameter class for plantation loblolly pine, Alabama, unit 5.

age class in Cohen (1986), the overall per tree growth average is affected by the frequency representation of the diameter classes. The mean growth rate for the population is influenced by the shifting of tree frequency distribution curves to the right. This shift is consistent with the increasing age, basal area, volume per acre, and growth rate and the analytical conclusions published for Alabama in Rudis et al. (1984).

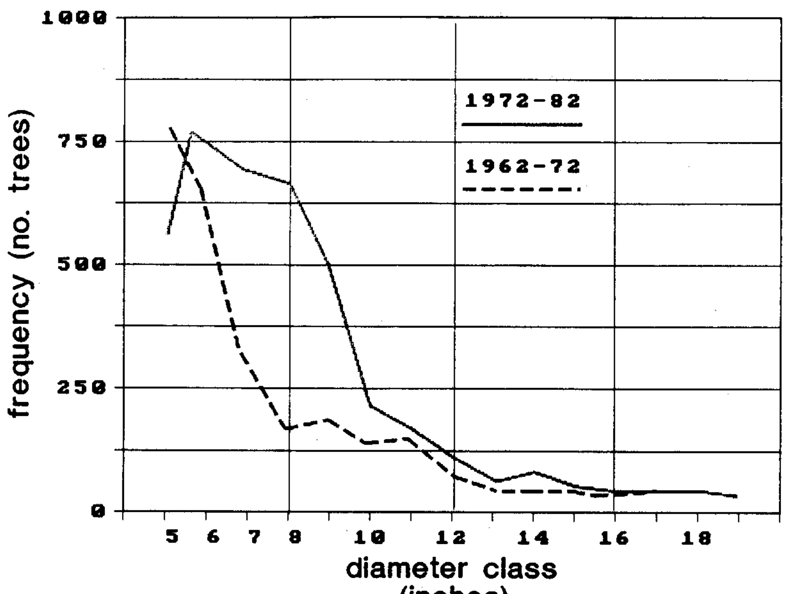

FIGURE 4. Population (frequency) distribution by diameter class for plantation loblolly pine, Alabama, unit 5.

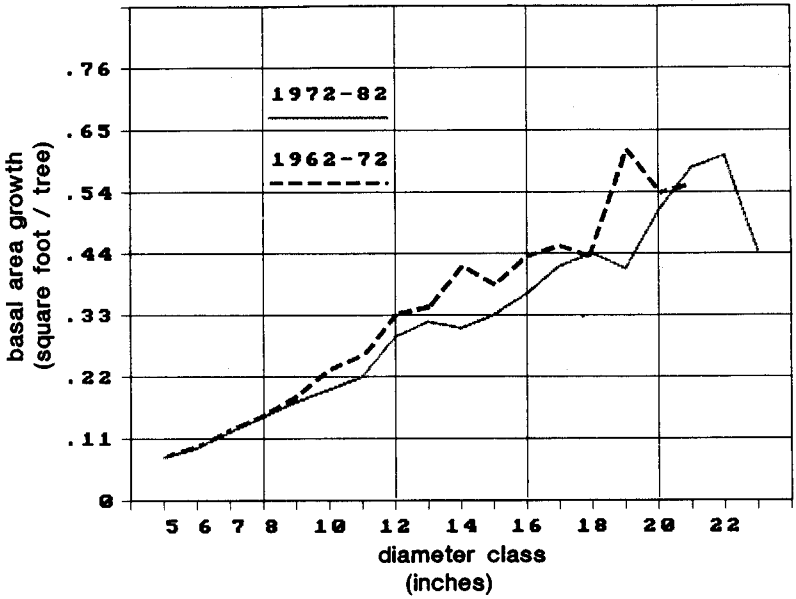

FIGURE 5. Basal area growth per tree by diameter class for natural loblolly pine, Alabama, unit 2.

CONCLUSIONS

Analysis of disaggregated data, whether growth rates, mortality rates, or other by-diameter-class data, can lead to paradoxical results. Frequencies of observed data cell means can cause the order of means to shift when unequal numbers of observations are associated with the cells at different times. Just

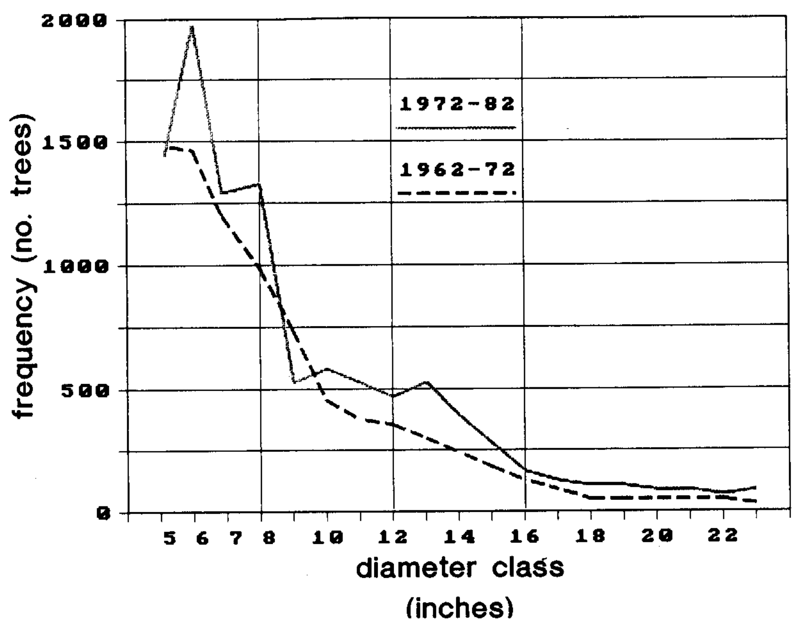

FIGURE 6. Population (frequency) distribution by diameter class for natural loblolly pine, Alabama, unit 2.

as there are cases where the order of means shifts, there are conditions for which no reordering of means will occur. To determine which condition is relevant, a careful analysis of frequency changes between cells must be undertaken.

Interpretation of the Alabama data with the awareness of Simpson's paradox reassures us that growth rates of loblolly in natural and plantation stands is not declining everywhere. In only two of the survey units in which loblolly is the major pine species was there an apparent slowdown in basal area growth, and in no case was there a statistically significant reduction.

Uncorrected radial growth rates in all four of the survey units were lower for the 1972–82 period than for the 1962–72 survey period. However, these results do not mean that all regions showing radial growth rate declines are actually declining in basal area growth rate. It is important that analyses of such data are done with a good deal of care and that results from one state (Georgia) should not be extrapolated to another (Alabama) on the basis of simple graphs showing radial growth rate by diameter class.

Analysis of survey growth data for large areas can be a risky undertaking, especially if the data come from horizontal point sampled plots. The tree-stand interaction over time is quite complex. Simple radial growth rates can be misleading. To avert the three problems identified in the original analysis of radial growth in Alabama survey units, one should: (1) use the initial probability of selection to obtain correct within-diameter class means, (2) translate the radial growth into basal area growth by diameter class, and (3) examine the population frequency distribution by diameter class. Awareness of Simpson's paradox and careful analysis following the steps outlined above can add to the inventory analyst's understanding of forest growth rates and stand processes.

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APPENDIX

Appendix Figures A1 and A2 are reproduced from (Cohen 1986) The American Statistician, February 1986, Vol. 40, No. 1, p. 33–34.

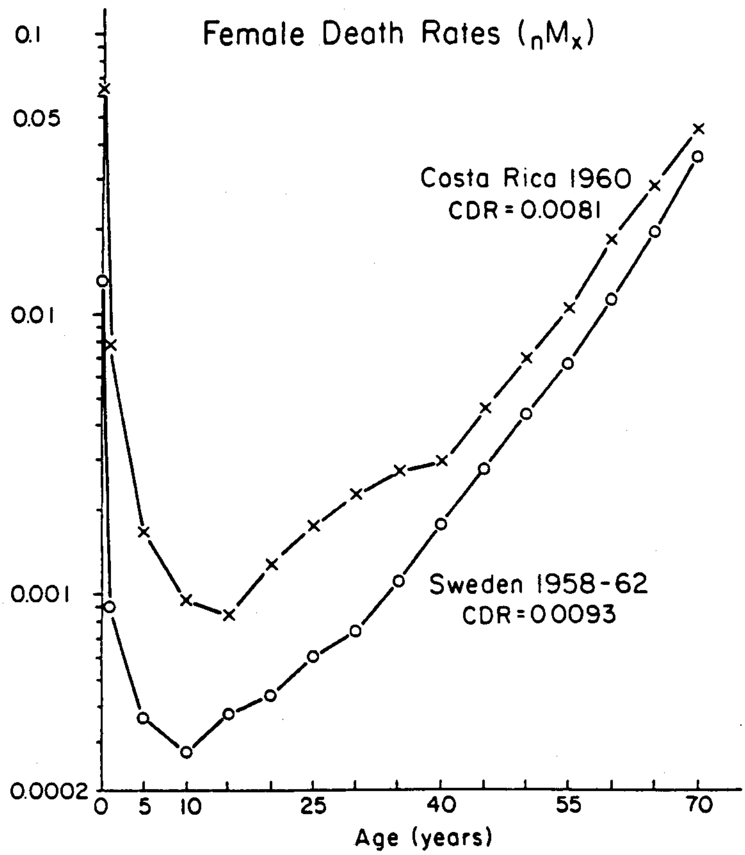

FIGURE A1. Age-specific female death rates of Costa Rica in 1960 and Sweden in 1958–1962. Source of data: Keyfitz and Flieger (1968). Though every age-specific death rate of Sweden is lower than the corresponding age-specific death rate of Costa Rica, the crude death rate (CDR) of Sweden exceeds that of Costa Rica.

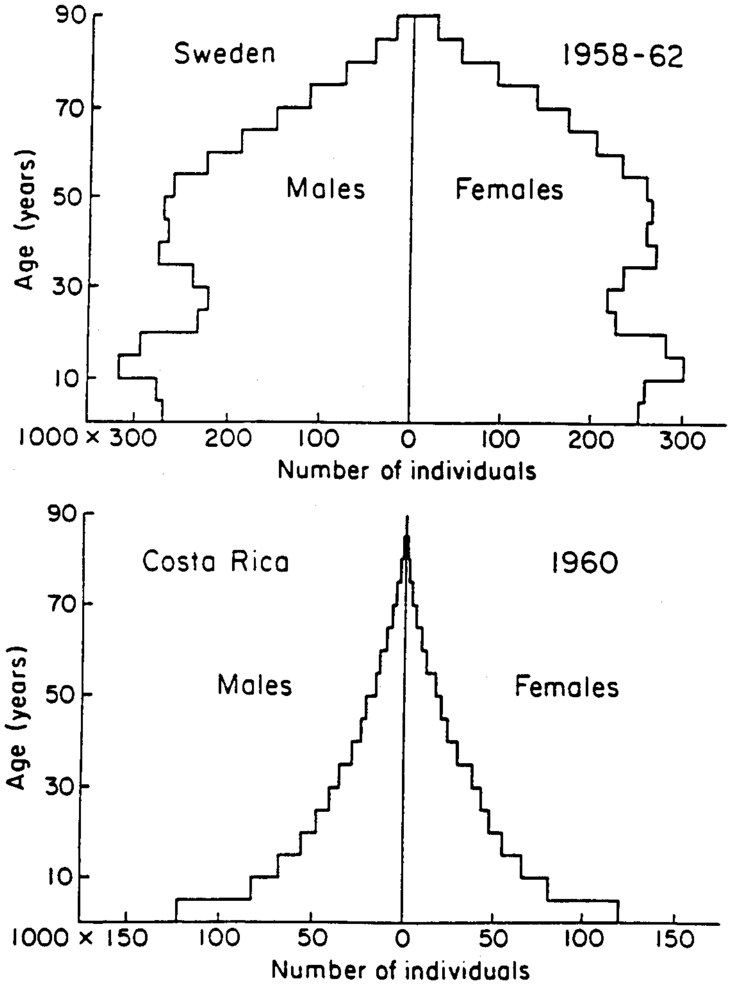

FIGURE A2. The age pyramids of Sweden in 1958–1962 and Costa Rica in 1960. Source of data: Keyfitz and Flieger (1968). The Costa Rican population has a much higher proportion of young individuals, whose death rates are less than those of old individuals in either Costa Rica or Sweden.